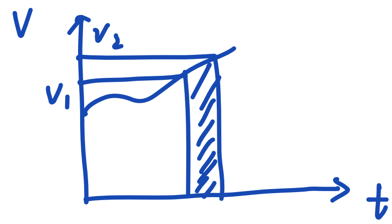
能量守恒：

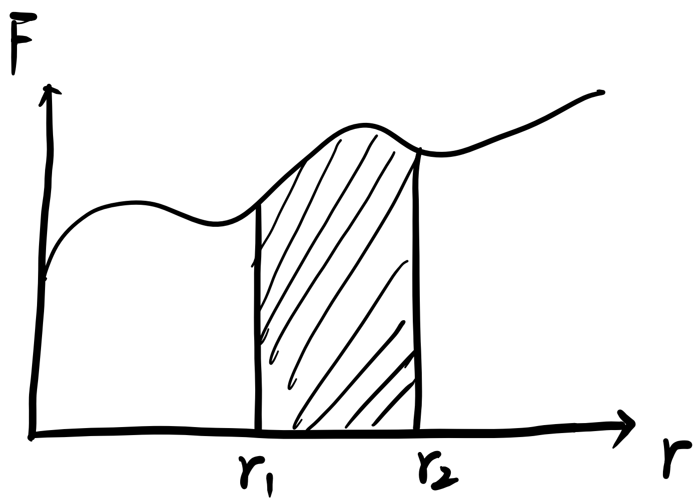
$$v_2^2 - v_1^2 = 2a(r_2 - r_1)$$



$$\frac{1}{2}m(v_2^2 - v_1^2) = ma(r_2 - r_1)$$

$$E_{k_2} - E_{k_1} = F\Delta r$$

动能的变量，外界做功。



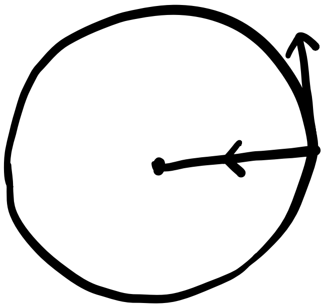
$\vec{F} \cdot d\vec{r}$ 做功微元

$\vec{F}(\vec{r})$

力在力的方向上

位移的积累

匀速圆周运动，功能原理



$$\Delta W = \int \vec{F} \cdot d\vec{r}$$

$$= \Delta E_k = 0$$

时时垂直

$$\int F\,dx = \int -kx\,dx = -\frac{1}{2}kx^2\Big|_{x_1}^{x_2}$$

$$= \frac{1}{2}k(x_1^2 - x_2^2)$$

$$E_{k2} - E_{k1} = \frac{1}{2}k(x_1^2 - x_2^2)$$

$$\Delta E_p = -\Delta E_k = -F\Delta x$$

$$\frac{1}{2}kx_1^2 + E_{k1} = \frac{1}{2}kx_2^2 + E_{k2}$$

机械能守恒.

非保守力

friction: $\int F_f\,dx$ (速度反向，F_f 反向) $\Rightarrow$

F_f 不是位置 X 的函数

（电磁相互作用）

$F = -kx$ (position dependent)

conservative force $\Rightarrow$

conservation

积分非路径依赖（前尘往事成云烟）

potential :

$$\int_A^B dE_p = \ominus \int_A^B F(x)\,dx$$

$$dE_p = -F\,dx$$

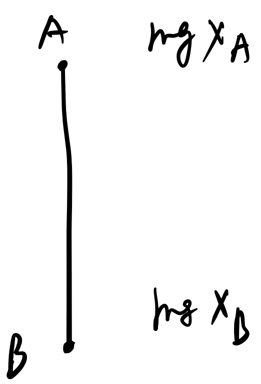
势能的功 对应势能减少

$$\Delta E_p = -F(x)\,\Delta x \qquad \vec{F}(x) = m\vec{g} \downarrow$$

$$E_{pB} - E_{pA} = -\int_A^B F(x)\,dx \qquad \text{(正向)}$$

自带符号

$$= +mg\,(x_B - x_A)$$



$$E_{pB} = mg\,x_B + C$$

$$E_{pA} = mg\,x_A + C$$

$$E_p(y) = -\int_0^y F\,dx$$

$$= mgy$$

约定 $x=0$ 处 势能为 0

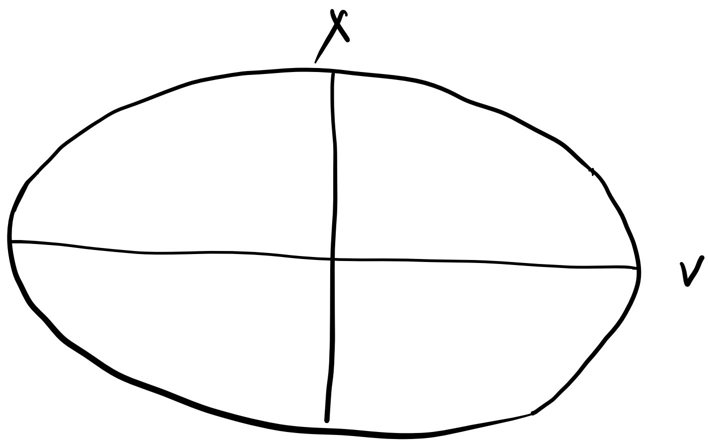
$$-\int_0^x F\,dx = \frac{1}{2}kx^2$$

变化量为 x，无论伸长或缩短，

弹性势能均为 $\frac{1}{2}kx^2$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \Rightarrow$$

$$\frac{v^2}{2E/m} + \frac{x^2}{2E/k} = 1$$



开普勒运动

谐振子

量子化条件

椭圆上每一点对应相同能量

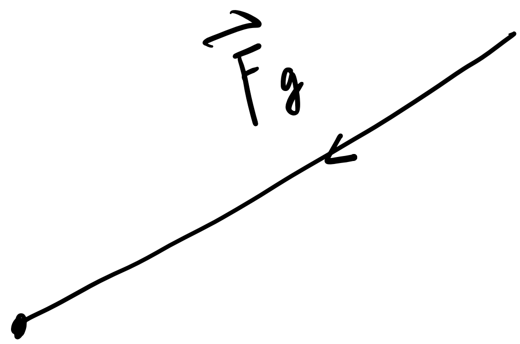
more general ： 做功 $\longrightarrow$ 动能改变

引力做功： 方向导数，引力势能

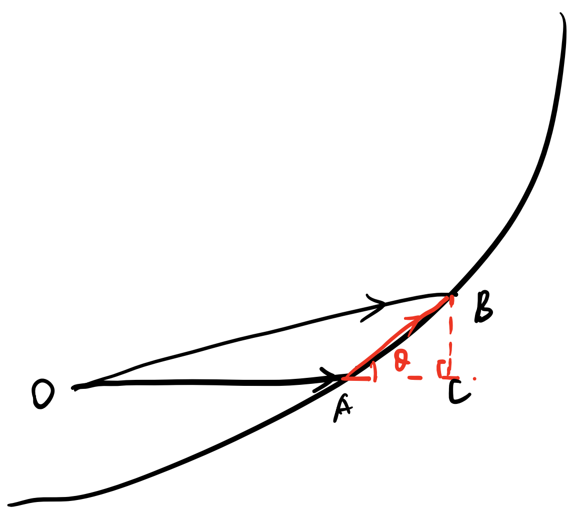
$\vec{F}_g = -\frac{GMm}{r^2} \vec{e}_r$ （吸引力）

问题：

（引力势能零点在哪？）



$\vec{F}_g = -\nabla E_p = -\frac{GMm}{r^2} \vec{e}_r$



几何

$\int dA = \int_A^B \vec{F}(r) \cdot d\vec{r}$

$$= \int_A^B - \frac{GMm}{r^2} \boxed{d\vec{r} \cos\theta}$$

(吸引力)

$\parallel$

dr

$B \to A$

$dr = |OB| - |OA| \approx$ in the limit, $OC = OB$

$|OC| - |OA| = |AC|$

$AC = |d\vec{r}| \cos\theta = dr$

$$= \int_A^B - \frac{GMm}{r^2} dr$$

$$= \int_A^B d\left(\frac{GMm}{r}\right)$$

$$E_p \Big|_A^B = - \int_A^B d\left(\frac{GMm}{r}\right)$$

代数处理

$$E_p^B - E_p^A = - \int_A^B \vec{F} \cdot d\vec{r}$$

$$= -\int_A^B (-)\frac{GMm}{r^2}\,\vec{e}_r \cdot d\vec{r}$$

$$= +\int_A^B \frac{GMm}{r^3}\left[r\,\vec{e}_r \cdot d\vec{r}\right]$$

$$=$$

$$\cancel{\left[(x, y, z) \cdot (dx, dy, dz)\right]}$$

$$x\,dx + y\,dy + z\,dz$$

$$=$$

$$\frac{1}{2}\,d(x^2+y^2+z^2) = \frac{1}{2}\,dr^2$$

$$= r\,dr$$

$$= +\int_A^B \frac{GMm}{r^2}\,dr$$

$$= \int_A^B d\left(-\frac{GMm}{r}\right)$$

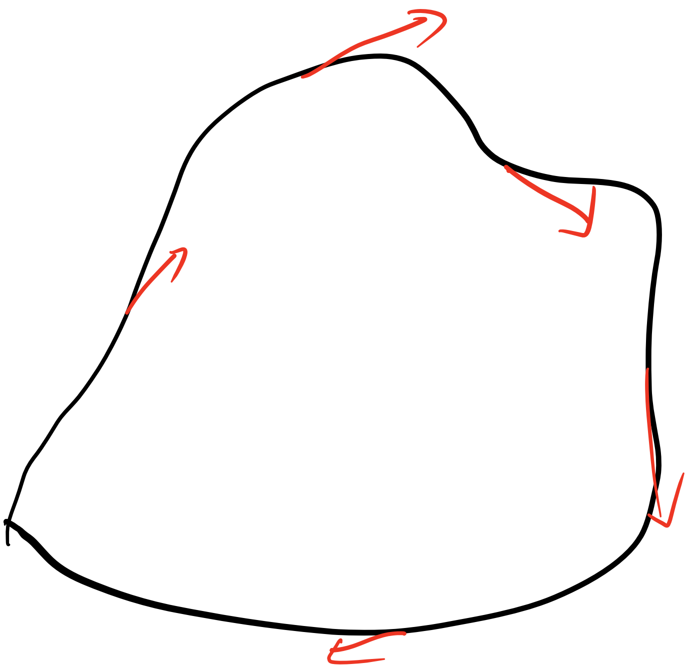
$$E_p = -\frac{GMm}{r}$$

非保守力

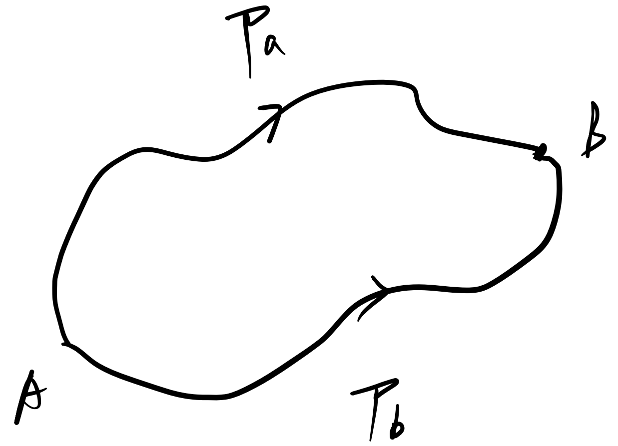
$$\int_A^B (F_c + \vec{F}_{nc})\,dx = \Delta W = \Delta K$$

$$W_{nc} + U(A) - U(B) = E_k(B) - E_k(A)$$

$$W_{nc} = \Delta(E_k + U)$$



$$\oint \vec{F_c} \cdot d\vec{r} = 0$$



$$\int_A \vec{F} \cdot d\vec{r} = \int_B \vec{F} \cdot d\vec{r}$$

$$\int_A \vec{F} \cdot d\vec{r} - \int_B \vec{F} \cdot d\vec{r}$$

$$= \oint \vec{F} \cdot d\vec{r} = 0$$

Relation between conservative force and potential energy

In 1D: $U(x_2) - U(x_1) = -\int_1^2 F_c \, dx$

$$F_c = -\frac{dU}{dx}$$

例： ① $U_G = mgz + \text{const}$

$$F_G = -\frac{dU_G}{dz} = -mg$$

② $U_S = \frac{1}{2}kx^2 + \text{const}$

$$F_S = -\frac{dU_S}{dx} = -kx$$

$\downarrow$

generalize to 3D

$$\vec{F}_c = \left(-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial z}\right)$$

$$F_x = -\frac{\partial U}{\partial x} = GMm\frac{\partial}{\partial x}\left(\frac{1}{r}\right)$$

$$= GMm\left(-\frac{1}{r^2}\right)\frac{\partial(x^2+y^2+z^2)^{\frac{1}{2}}}{\partial x}$$

$$= \sim \frac{\frac{1}{2}(2x)}{r}$$

$$F_y = \sim \frac{y}{r}$$

$$F_z = \sim \frac{z}{r}$$

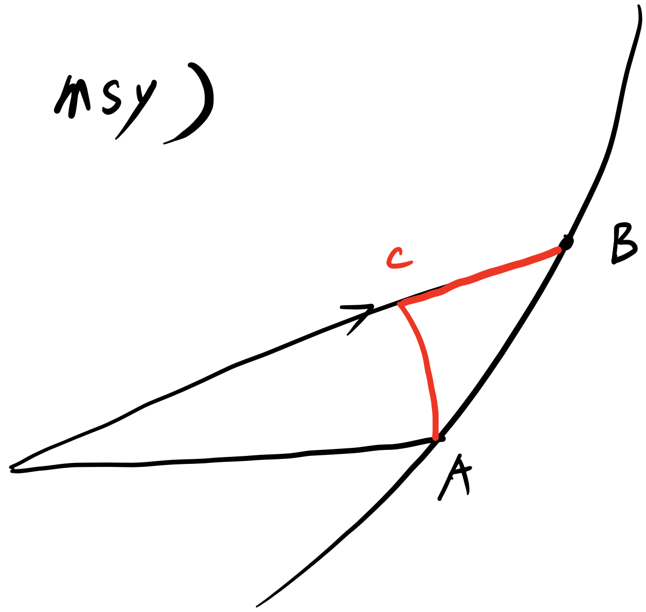
$$F = -\frac{GMm}{r^3}\underbrace{(x,\ y,\ z)}_{\text{坐标}}$$

$$= -\frac{GMm}{r^2}\vec{e}_r$$

坐标的沂分量确实方向

学生新思路

（强基 2020 MSY）



$$\Delta W = \int_A^B \vec{F} \cdot d\vec{r}$$

$$= \int_{ACB} \vec{F} \cdot d\vec{r}$$

$$= \underbrace{\int_A^C \vec{F} \cdot d\vec{r}}_{=0} + \underbrace{\int_C^B \vec{F} \cdot d\vec{r}}_{\text{径向}}$$

note：很多学生卡在这里，

$$W = \int_A^B \vec{F} \cdot d\vec{r}$$

$\vec{F} = m\vec{g} = -mg$ （竖直向下）

$A \to B$ 下降

如果是自由落体，$d\vec{r}$ 在一维情况下

写成 dr 是自带符号

依赖于所选路径，dr 有正有负

同理 $F = -kx$

x 是位移，x 本身带符号

我们处理引力的功问题：

① 几何

② 代数

③ 路径选择