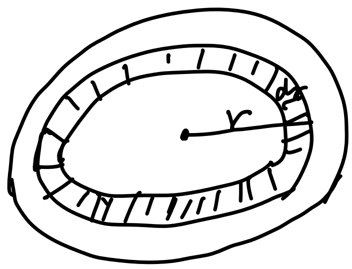
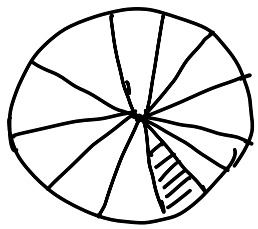


谢智凯
(XieZK)

面积只能用面积覆盖，不能用环来覆盖

$$dS = 2\pi r\,dr$$

$$S = \int_0^R 2\pi r\,dr = \pi R^2$$



长度量纲 ≠ 面积量纲
(SNX)

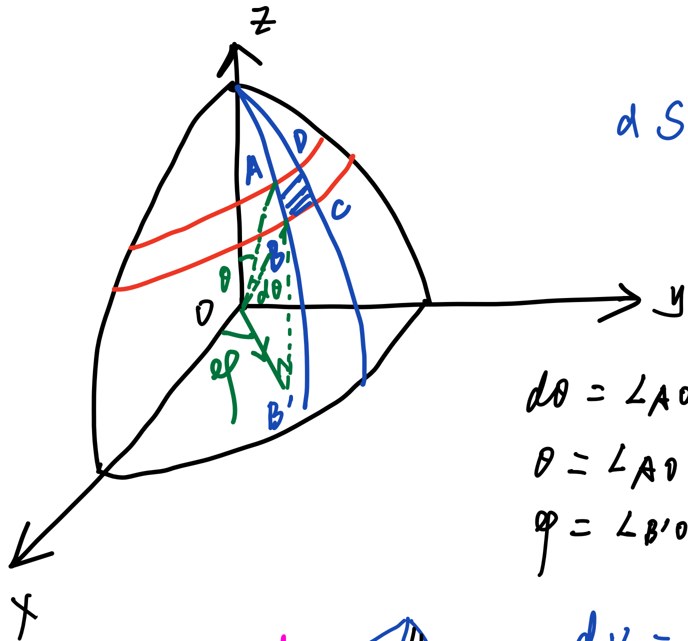
也不用线段覆盖
只能用小扇形覆盖

$$r\,d\theta\,dr = dS$$

$$\int_0^{2\pi}\int_0^R r\,d\theta\,dr = \pi R^2$$

同理，体积只能用体积覆盖
线段只能用线段覆盖，不能用点
（有理数填不满数轴）

体积微元：球坐标



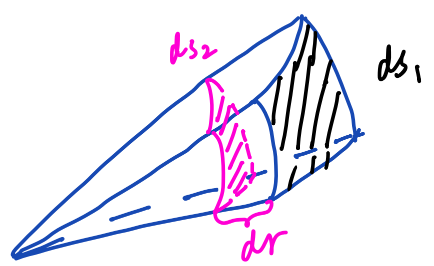
$$dS = |AB|\ |BC| = r\,d\theta\ r\sin\theta\,d\varphi$$

$d\theta = \angle AOB$

$\theta = \angle AOz$

$\varphi = \angle B'Ox$

$BB' \perp xoy$



$$dV = dr\,ds = r^2\sin\theta\,dr\,d\theta\,d\varphi = r^2\,dr\,d\Omega$$ 立体角

（内壳表面 ds_2 与外壳表面 ds_1 所夹住的体积）

极角

θ： polar angle

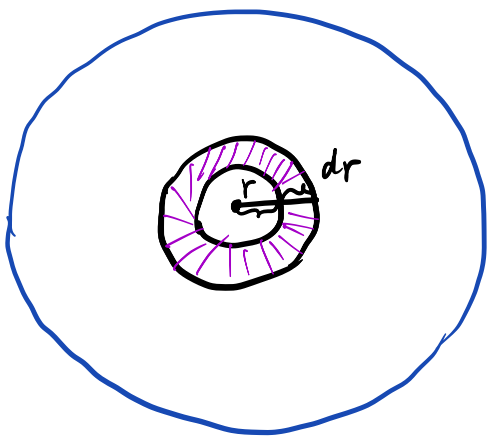
φ： azimuthal angle

方位角

问题：求一个球体的转动惯量，

如果是以下思路所得结果是 $I=\frac{3}{5}mR^2$，

哪里出错？（m：球质量，R：球半径）



黑色表示两个同心球壳，认为 r 从 0 到 R 构成一个球，

紫色部分为选取微元 dm

这样 $I = \int_0^R r^2\, dm$

$= \int_0^R r^2 \rho \underbrace{4\pi r^2}_{\text{球的表面积}} dr$

$= 4\pi\rho \left.\frac{r^5}{5}\right|_0^R$

$$m = \rho \times \frac{4}{3}\pi R^3$$

$$I = \frac{3}{5} m R^2$$

但是：正确答案是 $I = \frac{2}{5} m R^2$

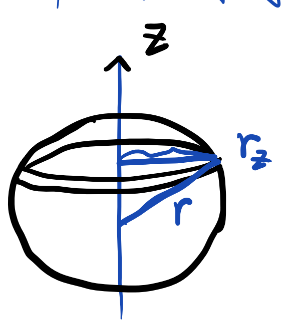
为什么呢？

$$I = \int_0^R r^2 \, dm$$

$$\overset{?}{=} \int_0^R r^2 \rho \underbrace{4\pi r^2}_{\text{球的表面积}} dr$$

问题在这！

转动惯量对于刚体而言，是相对于转动轴而言



$\boxed{r_z\text{，不是 } r\text{！}}$

到转轴距离，不是到球心

（转轴均过球心）

比如 相对于 z 轴，　$r_z^2 = x^2 + y^2$　①

同理　x 轴，　$r_x^2 = y^2 + z^2$　②

y 轴，　$r_y^2 = x^2 + z^2$　③

$r_x^2 + r_y^2 + r_z^2 = 2r^2$

球体各向同性

$$I_x = I_y = I_z$$

$$= \frac{1}{3}(I_x + I_y + I_z)$$

$$= \frac{1}{3} I_t$$

$$I_t = \int_0^R 2 r^2 \, dm$$

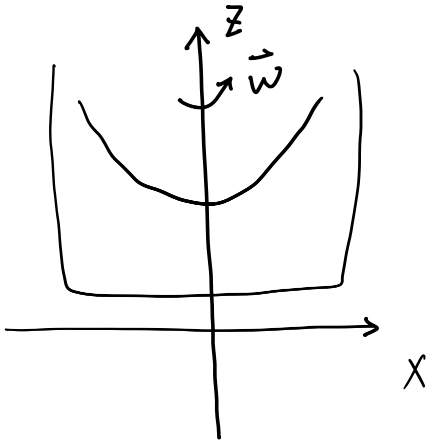
$$= 2 \times \frac{3}{5} mR^2$$

$$\therefore \quad I_z = 2 \times \frac{3}{5} \times \frac{1}{3} mR^2$$

$$= \frac{2}{5} mR^2$$

注：还可以用球坐标体积元直接计算 I_z

Exer:

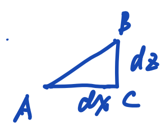


求液面高度对x的依赖关系？

郭昊葳

$a\,dx$ (等压面)

$$\boxed{dm}\,\omega^2 x = \cancel{\rho}\, g\, dz\, \cancel{ds}$$



$$= \cancel{\rho}\,\cancel{ds}\,dx$$

$P_C - P_B = \rho g\,dz$

$P_C - P_A = \rho a\,dx$

$P_A = P_B$

两个侧面的压力差提供向心加速度

$$\omega^2 x\,dx = g\,dz$$

$$z = \frac{\omega^2}{2g}x^2 + h_0$$

(抛物线)

$$\frac{dz}{dx} = \tan\theta = \frac{m\omega^2 x}{mg} = \frac{\omega^2 x}{g}$$

$$\frac{a}{g} = \frac{dz}{dx}$$

另：如何确定h_0？

Another acceleration:

Rotating bucket:

a curved surface

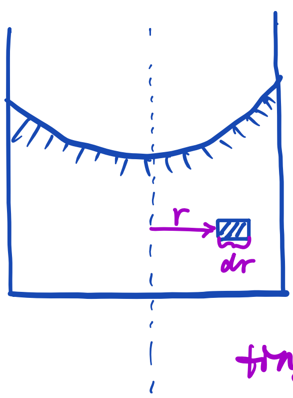
the same logic:

① The pressure field $p = p(r, z)$

② EOM in the r-direction is

$\longrightarrow$ 正方向

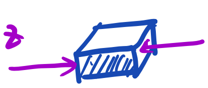
$$[-p(r+dr) + p(r)]\, dy\, dz = -dm \underset{}{\overset{\omega^2 r}{\overset{\parallel}{a}}}$$



$$-\frac{\partial p}{\partial r} \underbrace{\boxed{dr\, dy\, dz}}_{= dv} = -\rho r \omega^2 dr\, dy\, dz$$

$$\frac{\partial p}{\partial r} = \rho \omega^2 r \quad ①$$

tiny cube



$$\frac{\partial p}{\partial z} = -\rho g \quad ②$$

combine ①

$$p(r, z) = p_0 + \frac{1}{2}\rho \omega^2 r^2 - \rho g z$$

The surface is determined by the tension

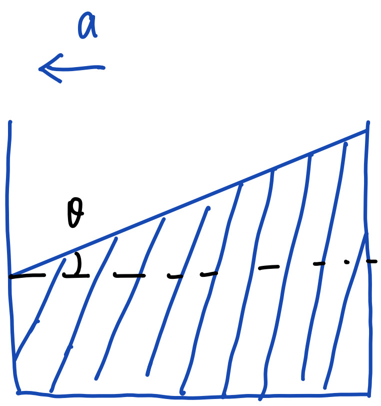
$$p(r, z) = p_0 \quad , \quad \text{i.e.}$$

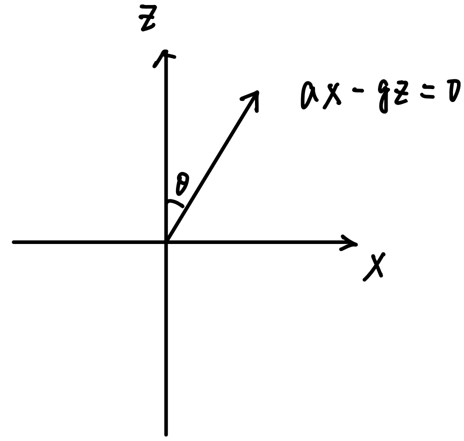
$$\frac{1}{2}\rho w^2 r^2 - \rho g z = 0 \implies$$

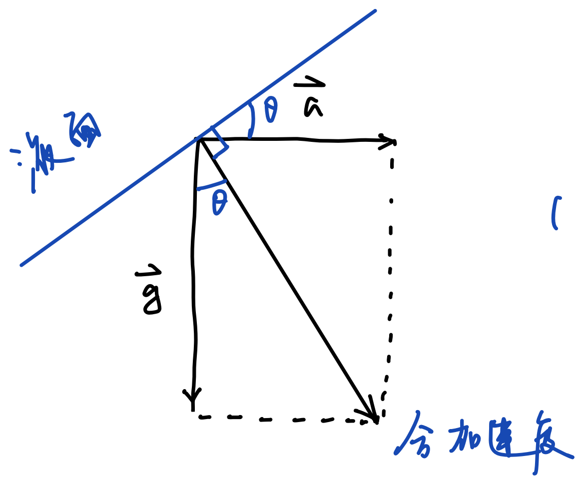
$$z = \left(\frac{w^2}{2g}\right) r^2$$

(paraboloid)

比较下面的平面







(否则会有加速分量)

$\tan\theta = a/g$

"Smart" pressure in liquids, where pressure is a scalar

$\vec{g}_{eff} = (a, 0, -g)$

$\nabla P = \rho \vec{g}_{eff}$

$\Delta P = \rho \vec{g}_{eff} \cdot d\vec{r}$

$P = P_0 + \rho a x - \rho g z$