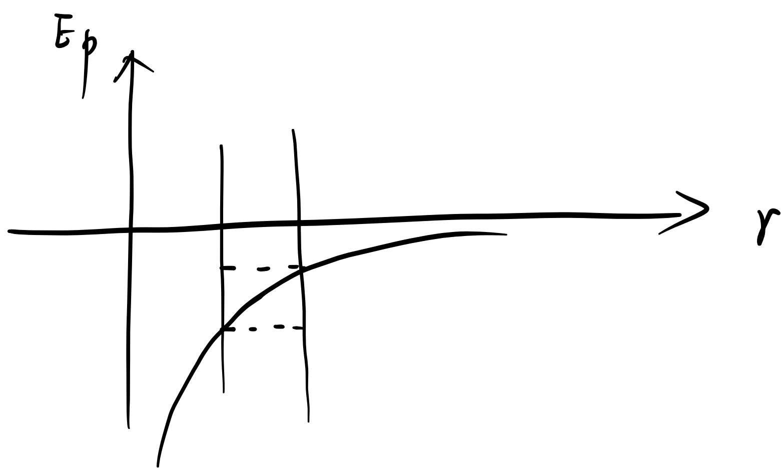
kepler's three laws

$$E_p = -\frac{GMm}{r}$$

$$E_p = mgh$$

矛盾吗?



$$E_p = -\frac{GMm}{(R_E + h)}$$

$$= -\frac{GMm}{R_E}\frac{1}{(1+\frac{h}{R_E})}$$

$$= \sim \left(1 - \frac{h}{R_E} + \dots\right)$$

$$= -\frac{GMm}{R_E} + \boxed{\frac{GMm}{R_E^2}}\, h \quad (= mg)$$

I. Each planet moves around the sun in an ellipse, with the sun at one focus.

II. The radius vector from the sun to the planet sweeps out equal areas in equal intervals of time.

III. The squares of the periods of any two planets are proportional to the cubes of the semimajor axes of their respective orbits: $T \propto a^{3/2}$.

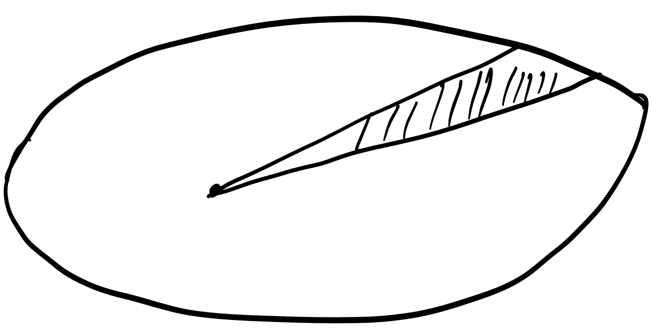
1st law : ellipse

轨道在以太阳为焦点的椭圆

平面上

2nd law : $\frac{dA}{dt} = \text{const}$

扫过的面积速度相等



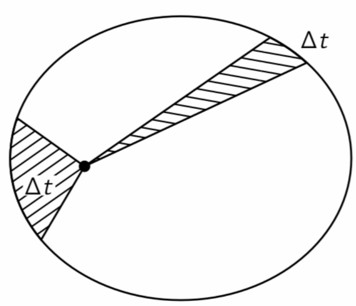
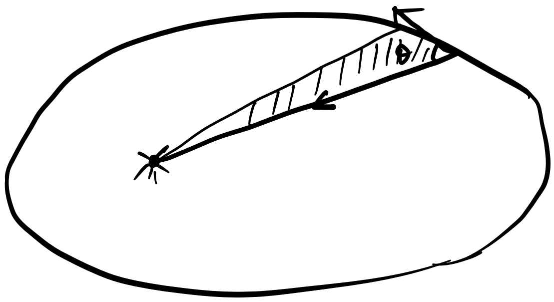


Fig. 7-2. Kepler's law of areas.



$$\vec{\tau} = \vec{r} \times \vec{F} = 0$$

$$\vec{L} = \vec{r} \times \vec{p} = C$$

A line drawn from the sun to a planet sweeps out equal areas in equal times

$$\underline{r\, m v \sin\theta} \quad dt$$

$$\boxed{r\, v\, dt \sin\theta}$$

$$\shortparallel$$

$$\boxed{2\,ds}\, m = c\, dt$$

$$\frac{ds}{dt} = \frac{c}{2m} = \text{const}$$

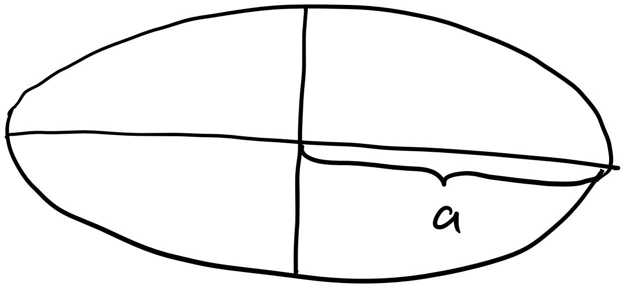
掠面速度 $= \frac{L}{2m}$

Kepler's 2nd law

Central force

$$G = 6.67 \times 10^{-11}\ m^3/kg \cdot s^2$$

3rd Law : $\frac{T^2}{a^3} = const$



if it is circle

$$\frac{GMm}{r^2} = m\frac{v^2}{r} = m\frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

=>

$$\frac{GM}{r^2} = \frac{4\pi^2 r}{T^2}$$

$$\frac{T^2}{r^3} = const = \frac{4\pi^2}{GM}$$

(1546 ~ 1601) danish astronomer (丹麦)

Tycho : rich guy, build lab

kepler : assistant

(1571 ~ 1630) 40 years' work

Newton : (1643 - 1727)

$$E = E_p + E_k$$

$$= -\frac{GMm}{r} + \frac{1}{2}mv^2$$

$$= -\frac{GMm}{r} + \frac{1}{2}m\frac{v^2}{r}r \quad \text{(圆周运动)}$$

↓ 万有引力提供向心力

$$= -\frac{GMm}{r} + \frac{1}{2}\frac{GMm}{r}$$

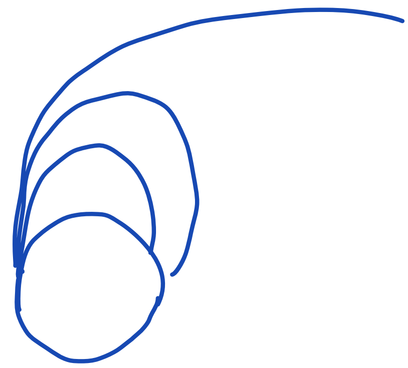
$$= -\frac{1}{2}\frac{GMm}{r}$$

$$E < 0$$

总能量为负 是不能到达无穷远处的

若 then, $E_p = 0$, $E_k < 0$

如何逃逸地球呢？



$E > 0$

$E = 0 \Rightarrow$

$$E_k = \frac{1}{2} m v^2 = \frac{GMm}{r} = -E_p$$

for the earth, $r = R_E$

$$v = \sqrt{\frac{2GM}{r}}$$

以这样的速度发射将不再回来，

V_1 : for the earth (~11.2 km/s)

V_2 : for the sun (~617.7 km/s)

V_3 : for the Galaxy

. . . .

for black hole (~3×10^5 km/s)

(what's the meaning?)

第一，二，三 escape velocity

第一宇宙速度书上一般称为贴地圆周运动

$$v = \sqrt{\frac{Gm}{r}}$$ （并未逃逸）

v_1 ： 俗称第二宇宙速度

$m_e = 6 \times 10^{24}\ kg$

地球质量

月球有大气层吗？

方均根速率 $v = \sqrt{\frac{3RT}{M}}$ ↓ mol质量 ($T \sim 20°C$)

H_2 : 1908 m/s
He : 1350 m/s
O_2 : 477 m/s
N_2 : 510 m/s
CO_2 : 407 m/s

} 宇宙中占大部分

重力吸收

气体分子

How to estimate these ?

原子很轻？ 只是密度小而已

$\frac{GMm}{r^2}$ 与 ρ 没关系。

For moon, $v_e = \sqrt{\frac{2GM_m}{r_m}} = 2400$ m/s

$M_m = 7.34 \times 10^{22}$ kg $\sim 0.0123 \, M_e$

$r_m = 1737$ km $\sim 0.273 \, r_e$