Feynman Lectures (I)

The simplest change to observe in a body is the apparent change in its position with time

Table 8-2

t (sec)	s (ft)
0	0
1	16
2	64
3	144
4	256
5	400
6	576

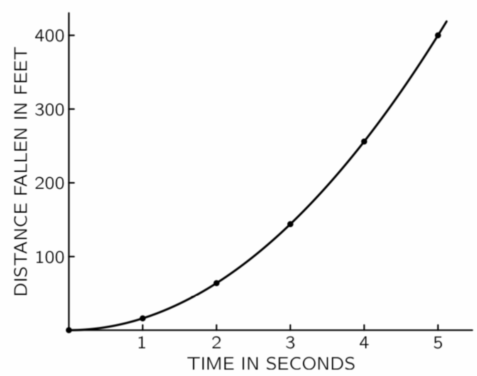


Fig. 8-2. Graph of distance versus time for a falling body.

speed as a derivative

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

" derivative of s with respect to t "

distance as an integral

$$S = \Sigma\, v\, \Delta t$$

$\downarrow$

(the Greek letter Σ (sigma) is used to denote addition)

$$S = \sum_i v(t_i)\, \Delta t$$

$$t_{i+1} = t_i + \Delta t$$

not correct because the velocity

changes during the time interval Δt

$$S = \lim_{\Delta t \to 0} \sum_i v(t_i)\, \Delta t$$

$\Delta \to d$ (infinitesimal)

$v(t_i) \longrightarrow$ instant velocity

the addition is written as a sum with

a great "s", $\int$ (from the Latin summa)

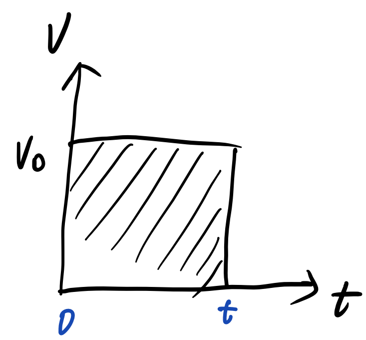
$$S = \int v(t)\, dt$$

Acceleration

$$a = \frac{dv}{dt}$$

e.g. 1d , 匀速

$$V = V_0 = \text{const}$$



derivative $\frac{dv}{dt} = a = 0$

integra $r = \int_0^t v(t)\,dt = V_0 \int_0^t dt = V_0 t$

匀变速：自由落体

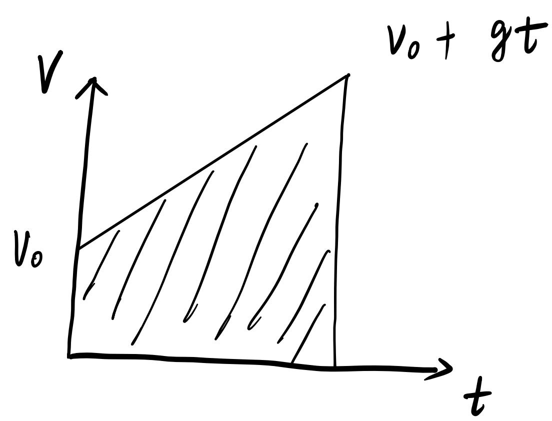
free fall

v_0, r_0, g, t $(t_0 = 0)$

$r(t) = ?$

$v(t) = v_0 + gt$

$$\int_0^t v(t) = v_0 t + \frac{1}{2} g t^2$$



$$S = \int_0^t v(t)\,dt = \frac{1}{2}(v_0 + v_0 + gt)\,t$$

$$= v_0 t + \frac{1}{2} g t^2$$

$$= \left[(v_0 + gt)^2 - v_0^2\right] / 2g$$

general Cases :

Variables : $\vec{r}$, $\vec{v}$, $\vec{a}$, t

components of vectors :

v_x, v_y, v_z

a_x, a_y, a_z

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

分量对应相等

数学基础： $\hat{x} \cdot \hat{y} = \hat{y} \cdot \hat{z} = \hat{x} \cdot \hat{z} = 0$

$\longleftarrow$

$$\vec{r} = \int \vec{v}(t)\, dt$$ indefinite

$$\vec{v} = \int \vec{a}(t)\, dt$$

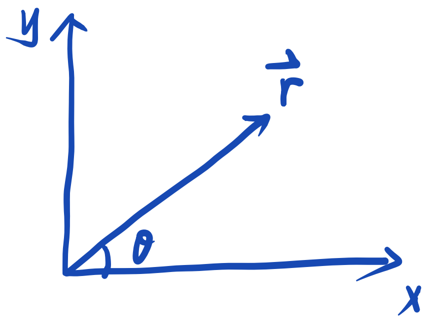
定积分： 给定初始条件

给定积分上下限

请大家回顾二维极坐标 (r, θ)，自己通过代数（几何）的方式推导

$$\frac{d\hat{r}}{dt} = \frac{d\theta}{dt}\hat{\theta}$$

$$\frac{d\hat{\theta}}{dt} = -\frac{d\theta}{dt}\hat{r}$$



$$\vec{r} = (x, y)$$
$$= (r\cos\theta, r\sin\theta)$$

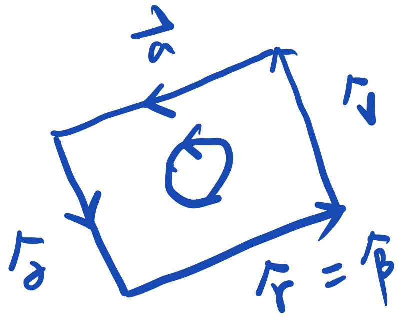
$$\vec{v} = \frac{d\vec{r}}{dt} = wr(-\sin\theta, \cos\theta)$$

$$\vec{a} = \frac{d\vec{v}}{dt} = w^2 r(-\cos\theta, -\sin\theta)$$

$$\vec{\partial} \equiv \frac{d\vec{a}}{dt} = w^3 r(\sin\theta, -\cos\theta)$$

$$\vec{\beta} \equiv \frac{d\vec{\partial}}{dt} = w^4 r(\cos\theta, \sin\theta)$$

用时间的一次导数连接



4次一循环

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{i\frac{\pi}{2}} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$$

$$i^2 = -1, \quad i^4 = 1$$

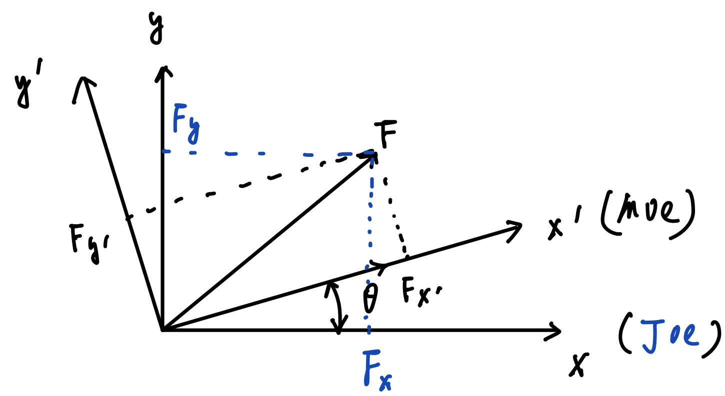
$\hat{\beta} = \hat{r}$，四次一个循环

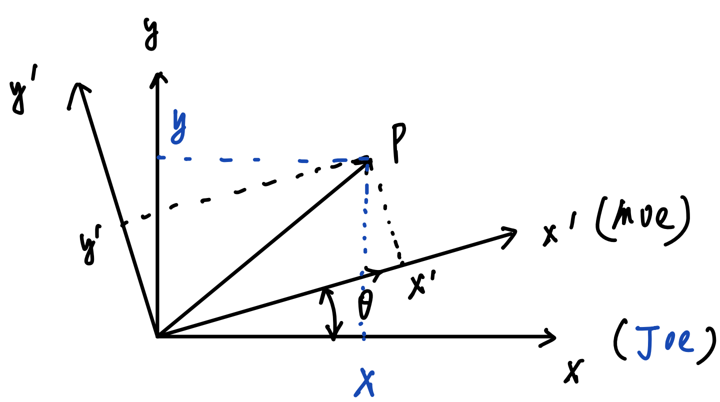
$$\frac{d\,e^{i\theta(t)}}{dt} = i\,e^{i\theta(t)}\frac{d\theta}{dt}$$

$$= e^{i(\theta(t)+\frac{\pi}{2})}\frac{d\theta}{dt}$$

矢量方向求一次时间的导数逆时针旋转 $\frac{\pi}{2}$

$\vec{F}$ is a vector ?





$$x' = x\cos\theta + y\sin\theta$$

$$y' = -x\sin\theta + y\cos\theta$$

$$z' = z$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix}$$

Joe $\rightarrow$ Moe

observer

identical form of transformation

$$\vec{F} = m\vec{a} = m\frac{d^2\vec{x}}{dt^2}$$

$$m\left(d^2x'/dt^2\right) = m\left(d^2x/dt^2\right)\cos\theta + m\left(d^2y/dt^2\right)\sin\theta$$

$$m\left(d^2y'/dt^2\right) = -m\left(d^2x/dt^2\right)\sin\theta + m\left(d^2y/dt^2\right)\cos\theta$$

$$m\left(d^2z'/dt^2\right) = m\left(d^2z/dt^2\right)$$

check $\vec{F}' = m\vec{a}'$

$$\vec{F}' = \begin{pmatrix} \cos\theta & \sin\theta & \\ -\sin\theta & \cos\theta & \\ & & 1 \end{pmatrix}\vec{F}$$

$$\vec{F} = m\vec{a}$$

$$\frac{d^2\vec{r}'}{dt} = \begin{pmatrix} \cos\theta & \sin\theta & \\ -\sin\theta & \cos\theta & \\ & & 1 \end{pmatrix} \frac{d^2\vec{r}}{dt^2}$$

$$\vec{F}' = m\,\frac{d^2\vec{r}'}{dt^2}$$

If Newton's laws are correct on one set of axes, they are also valid on any other set of axes.