wiki :

$$\begin{pmatrix} 1 & 2 & 3 \\ \sigma(1) & \sigma(2) & \sigma(3) \end{pmatrix}$$

rearrangement of 1 2 3

$$\begin{pmatrix} 1 & 2 & 3 \\ \sigma_a(1) & \sigma_a(2) & \sigma_a(3) \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ \sigma_b(1) & \sigma_b(2) & \sigma_b(3) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 \\ \sigma_{ab}(1) & \sigma_{ab}(2) & \sigma_{ab}(3) \end{pmatrix}$$

$$\sigma_{ab}(1) = \sigma_a(\sigma_b(1))$$

$$\therefore \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

$1 \to 3 \to 2$

$2 \to 1 \to 1$

$3 \to 2 \to 3$

保持box $\overset{1}{\square}$ $\overset{2}{\square}$ $\overset{3}{\square}$ 位置不变，沿着乘法顺序，保持出发的盒子编号，更换不同编号的粒子（successively）。

$\alpha \quad \beta \quad \gamma$: wave function

展开 $\begin{pmatrix} 1 & 2 & 3 \\ \sigma(1) & \sigma(2) & \sigma(3) \end{pmatrix} \alpha(1)\,\beta(2)\,\gamma(3)$

$$= \alpha(\sigma(1))\,\beta(\sigma(2))\,\gamma(\sigma(3))$$

交换粒子的编号，将状态 α, β, γ 顺序排开，使用连乘将非常方便

box : wave function

object : particle

$$\begin{pmatrix} 1 & 2 & 3 & \cdots \\ \alpha_1 & \alpha_2 & \alpha_3 & \cdots \end{pmatrix} = \begin{pmatrix} 3 & \boxed{\begin{matrix} 1 \\ \alpha_1 \end{matrix}} & 2 & \cdots \\ \alpha_3 & & \alpha_2 & \cdots \end{pmatrix}$$

重要的是匹配关系.

即 $S_3 \simeq D_3$ 中 $C = FB$

e.g. $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 3 & 2 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$